Chapter 7

Conclusions and Future Work

In the context of linear elasticity, a method has been established which incorporates local discontinuous functions into standard finite element approximations. An important feature of the method concerns the direct enhancement of the approximating space with discontinuous enrichment functions, such that the present formulation does not fall into the assumed strain category. The enriched basis is formed from a union of the nodal shape functions associated with the mesh and the product of nodal shape functions with discontinuous functions locally about a specified geometry. This construction allows for the modeling of discontinuities whose geometries are independent of the finite element mesh. The enriched approximation is substituted directly into the bilinear form, resulting in a linear algebraic system of equations with unknowns consisting of both classical and enriched degrees of freedom. The multiplication of the enrichment functions with local shape functions ensures a conforming approximation, as well as the maintenance of a measure of sparsity in the system of equations. All of the above features provide the method with distinct advantages over both standard finite element and meshless methods for modeling arbitrary discontinuities.

The method was easily applied to fracture mechanics with the additional incorporation of near crack-tip functions. In conjunction with the discontinuous enrichment, the near-tip functions allowed for the seamless modeling of the displacement discontinuity along the entire crack geometry. They also provided for accurate stress intensity factors with relatively coarse meshes. In the context of Mindlin-Reissner plate fracture, a domain form of the interaction integral was developed to extract the mixed moment and shear force intensity factors. While a degree of success was obtained by enriching a well-established plate formulation, certain adverse effects were observed for very thin plates and in the calculation of the shear force intensity factor. These effects, while relatively minor, suggest that it may be difficult to construct an enriched approximation which is completely free of shear locking.

A very general framework has been developed to treat non-linear constitutive laws on the discontinuous interface. The iterative scheme of the LATIN method which separates the local non-linearities from the global equations is seen to work naturally with the local discontinuous enrichment. For enforcing contact conditions on the interface, the method possesses several advantages over more common penalty techniques. The conditioning of the system of equations is not a concern, and the iterative process ensures that the contact conditions are satisfied to within a specified error tolerance. While unilateral and frictional contact was considered in this work, the framework allows for general constitutive laws on the interface. In the context of fracture mechanics, the modeling of cohesive laws and the utilization of the space-time principle of the LATIN method are possible areas for future research. Other areas for possible research include the modeling of localization bands and damage.

In addition to the suggestions already provided in previous chapters, it is hoped that the work presented in this thesis will provide a solid foundation for research in the field of fracture mechanics. At the present time, several other related areas of research suggest themselves. The extension to three dimensions is conceptually straightforward, although the interaction between the mesh and crack geometries is considerably more complicated. A key advantage of the FEM over the boundary element method (BEM) is the ability to model nonlinear material laws. The application of the present method to ductile fracture is therefore an area of considerable promise. The modeling of crack propagation along a bimaterial interface presents several challenges for traditional finite element methods, as both the geometries of the crack and material interface must be respected by the mesh. The discontinuous enrichment developed in this work circumvents one difficulty, and it is also suggested that appropriate enrichment functions can be developed to model the bimaterial interface.

In many of the examples presented in this thesis, the domain was partitioned with a uniform mesh. In addition to the ease of construction, a uniform mesh provides additional opportunities. The construction of approximation spaces with greater continuity is relatively straightforward on a uniform grid. Consider for example Hermite polynomials or cardinal splines, which are both easily constructed on a uniform mesh. In Donning and Liu (1998), an approximation was constructed using cardinal splines on a uniform mesh to avoid shear locking in beams and plates. With the addition of discontinuous enrichment, it is suggested that these methods can be naturally extended to model the fracture of such structures. One example is the fracture of plates in Kirchhoff theory, in which the solution space requires an approximation with a high degree of continuity. Non-uniform boundaries can be treated with this approach by only integrating the bilinear form on the domain Ω, which may or may not coincide with the mesh. This approach was in fact adopted in a limited sense in Donning and Liu (1998).